Math 10B with Professor Stankova
Quiz 6; Tuesday, 2/27/2018
Section #203; Time: 9:30 AM
GSI name: Roy Zhao **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** The PMF function f goes from subsets of $\mathbb{R}$ to $[0, 1]$.

Solution: The function f goes directly from $\mathbb{R}$ to $[0, 1]$, not subsets.

2. **TRUE** False The PMF function f is defined for all of $\mathbb{R}$ but is only nonzero on the range of X.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (3 points) I am conducting a survey on teenagers under 18. Suppose that there are an equal number of people aged 13 through 18, which is the population I am surveying. Let X be the age of the person I interview. Find and draw the PMF associated to X.

Solution: This is a uniform distribution from 13 to 18. There are 6 numbers in between and hence

$$f(k) = \begin{cases} \frac{1}{6} & x \in \{13, 14, 15, 16, 17, 18\} \\ 0 & \text{otherwise} \end{cases}.$$

(b) (4 points) Suppose that I may interview the same person twice. I interview 100 people total. What is the probability that I interview 30 or 31 middle schoolers (aged 13, 14)?

Solution: The probability of success is $f(13) + f(14) = \frac{2}{6} = \frac{1}{3}$. We can interview the same person twice so this is independent trials and hence binomial distribution. Then this probability is the probability that I interview 30 of them plus the probability that I interview 31 of them. So the probability is

$$f(30) + f(31) = \binom{100}{30} \left(\frac{1}{3}\right)^{30} \left(\frac{2}{3}\right)^{70} + \binom{100}{31} \left(\frac{1}{3}\right)^{31} \left(\frac{2}{3}\right)^{69}.$$

(c) (3 points) What is the probability that I have to interview 8 people until I interview a high schooler (aged $15-18$, the high schooler is the 8th person I interview)?

Solution: This is geometric. The probability of success is $f(15)+f(16)+f(17)+f(18)=\frac{4}{6}=\frac{2}{3}$. Plugging this into the geometric distribution gives

$$f(8)=\left(\frac{1}{3}\right)^7\left(\frac{2}{3}\right).$$